KEK-TH-1303

Classically conformal $B-L$ extended Standard Model

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Abstract

Under a hypothesis of classically conformal theories, we investigate the minimal $B-L$ extended Standard Model, which naturally provides the seesaw mechanism for explaining tiny neutrino masses. In this setup, the radiative gauge symmetry breaking is successfully realized in a very simple way: The $B-L$ gauge symmetry is broken through the conformal anomaly induced by quantum corrections in the Coleman-Weinberg potential. Associated with this $B-L$ symmetry breaking, the Higgs mass parameter is dynamically generated, by which the electroweak symmetry breaking is triggered. We find that a wide range of parameter space can satisfy both the theoretical and experimental requirements.

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Introduction.— The stability of the electroweak scale, in other words, the gauge hierarchy problem is one of the most important issues in the Standard Model (SM), which has been motivating us to seek new physics beyond the SM for decades. The problem originates from the quadratic divergence in quantum corrections to the Higgs self energy, which should be canceled by the Higgs mass parameter with extremely high precision when the cutoff scale is much higher than the electroweak scale, say, the Planck scale. The most popular new physics scenario which offers the solution to the gauge hierarchy problem is the supersymmetric extension of the SM where no quadratic divergence arises by virtue of supersymmetry.

Because of the chiral nature of the SM, the SM Lagrangian at the classical level possesses the conformal invariance except for the Higgs mass term closely related to the gauge hierarchy problem. Bardeen has argued [1] that once the classical conformal invariance and its minimal violation by quantum anomalies are imposed on the SM, it can be free from the quadratic divergences and hence the gauge hierarchy problem. If the mechanism really works, we can directly interpolate the electroweak scale and the Planck scale.

As was first demonstrated by Coleman and Weinberg [2] for the U(1) gauge theory with a massless scalar, the classical conformal invariance is broken by quantum corrections in the Coleman-Weinberg effective potential and the mass scale is generated through the dimensional transmutation. It is a very appealing feature of this scheme that associated with this conformal symmetry breaking, the gauge symmetry is also broken and the Higgs boson arises as a pseudo-Nambu-Goldstone boson whose mass has a relationship with the gauge boson mass and hence predictable (when only the gauge coupling is considered).

It was tempting to apply this Coleman-Weinberg mechanism to the SM Higgs sector, and the upper bound on the Higgs boson mass was found as $m_h \lesssim 10$ GeV under the assumption for a top quark mass $m_t \lesssim m_Z$ [2]. Unfortunately, this possibility is already excluded by the current experimental bound on the Higgs boson mass $m_h > 114.4$ GeV [3] and the top quark mass measurement $m_t = 172.4$ GeV [4]. In addition, there is an obvious theoretical problem that the Coleman-Weinberg effective potential in the SM becomes unbounded from below for $m_t > m_Z$ [5]. Therefore, in order to pursue this scheme, it is necessary to extend the SM. In fact, it has been found [6, 7, 8, 9, 10] that when the SM is extended to include an additional scalar, phenomenologically viable models can be obtained.

The reason that the Higgs boson mass was predicted to be lighter than 10 GeV in the original Coleman-Weinberg model is as follows. The effective potential in a classically conformal theory is given by

$$V_{eff} = \frac{\lambda h^4}{4} + Bh^4 \left(\ln \left(\frac{h^2}{\langle h \rangle^2} \right) - \frac{25}{6} \right) \tag{1}$$

where the renormalization scale is taken at $\langle h\rangle$ and the coefficient B for the SM is given by

$$B = \frac{3}{64\pi^2}\left(\frac{3g^4 + 2g^2g'^2 + g'^4}{16} - g_t^4\right) \tag{2}$$

where g, g' and g_t are the gauge couplings and the top Yukawa coupling, respectively. Here we have neglected the contributions of the other fields including the scalar itself. The coupling constant is renormalized as $V^{(4)}|_{h=\langle h\rangle} = 6\lambda$. Then the effective potential must satisfy $V'|_{h=\langle h\rangle} = 0$ and the renormalized coupling constant λ is related to B as

$$\lambda = \frac{44}{3}B. \tag{3}$$

The Higgs boson mass is given by

$$m_h^2 = V_{eff}^{(2)}|_{h=\langle h\rangle} = 8B\langle H\rangle^2 = \frac{6}{11}\lambda\langle h\rangle^2. \tag{4}$$

In order for the perturbative calculation to be valid, the coupling constant λ must be balanced with the 1-loop coefficient B. If the top Yukawa coupling were neglected, the dominant contribution to B would come from the gauge couplings and hence λ could not be so large. This gave the upper bound for the Higgs boson mass in the original Coleman-Weinberg scenario. At present we know that because of the large top quark contribution B is negative and the Coleman-Weinberg potential does not have a stable minimum with a positive m_h^2. The relation (4) should be compared with a similar relation for the classical potential with a negative mass squared term ($\mu^2 < 0$)

$$V = \frac{\lambda}{4}h^4 + \frac{\mu^2}{2}h^2. \tag{5}$$

In this case, the Higgs boson mass is given by

$$m_h^2 = 2|\mu^2| = 2\lambda\langle h\rangle^2, \tag{6}$$

and the coefficient is about 4 times larger than (4). Furthermore the coupling constant λ can become relatively larger than before so long as it does not diverge up to the ultra-violet cutoff scale, which we take at the Planck scale.

If both of the classical mass term μ^2 and the radiative corrections B are present, the relation (3) is modified to

$$\lambda = \frac{44}{3}B - \frac{\mu^2}{\langle h\rangle^2} \tag{7}$$

and the Higgs boson mass is given by

$$m_h^2 = \frac{6}{11}\lambda\langle h\rangle^2 + \frac{16}{11}|\mu^2| = \left(2\lambda - \frac{64}{3}B\right)\langle h\rangle^2. \tag{8}$$

This generalizes the two extremal cases (4) and (6).

The current experimental observations, in particular, the solar and atmospheric neutrino oscillations [11] motivate us to extend the SM so as to incorporate the neutrino masses and flavor mixings. One promising scenario is to introduce the right-handed neutrinos and their heavy Majorana masses, so that the tiny neutrino masses can be naturally explained by the seesaw mechanism [12]. By imposing the classical conformal invariance, the SM with the right-handed neutrinos has been recently investigated [10]. To keep the classical conformal invariance, a SM singlet scalar is introduced whose vacuum expectation value (VEV) generates masses of the right-handed neutrinos. Through numerical analysis, it has been shown that a set of parameters leads to phenomenologically viable results.

In this letter, we investigate a $B-L$ gauged extension of the SM with a classical conformal invariance. In addition to the SM particles, the model has the right-handed neutrinos ν_R^i and a SM singlet scalar Φ whose VEV gives the Majorana mass terms for ν_R^i and also breaks the $B-L$ gauge symmetry. Through an interaction with the scalar Φ field, Higgs boson also acquires the VEV and the electroweak symmetry is radiatively broken. Nevertheless, the dynamics of the SM Higgs sector is governed by the induced negative mass squared term, and if we impose the triviality and the stability bounds on the Higgs potential, the Higgs boson mass is bounded in the same range 130 GeV$\lesssim m_h \lesssim$ 170 GeV as the ordinary bounds in the SM [13]. The phenomenological constraint from the neutrino mass gives an upper bound for the $B-L$ breaking scale.

Model.— The model we will investigate is the minimal $B-L$ extension of the SM [14] with the classical conformal symmetry. The $B-L$ (baryon minus lepton) number is a unique anomaly free global symmetry that the SM accidentally possesses and can be easily gauged. Our model is based on the gauge group $\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y\times\mathrm{U}(1)_{B-L}$ and the particle contents are listed in Table 1 [15]. Here, three generations of right-handed neutrinos (ν_R^i) are necessarily introduced to make the model free from all the gauge and gravitational anomalies. The SM singlet scalar (Φ) works to break the $\mathrm{U}(1)_{B-L}$ gauge symmetry by its VEV, and at the same time generates the right-handed neutrino masses.

The Lagrangian relevant for the seesaw mechanism is given as

$$\mathcal{L} \supset -Y_D^{ij}\overline{\nu_R^i}H^\dagger \ell_L^j - \frac{1}{2}Y_N^i \Phi \overline{\nu_R^{ic}}\nu_R^i + \text{h.c.}, \tag{9}$$

where the first term gives the Dirac neutrino mass term after the electroweak symmetry breaking, while the right-handed neutrino Majorana mass term is generated through the second term associated with the $B-L$ gauge symmetry breaking. Without loss of generality, we here work on the basis where the second term is diagonalized and Y_N^i is real and positive.

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-L}$
q_L^i	$\mathbf{3}$	$\mathbf{2}$	$+1/6$	$+1/3$
u_R^i	$\mathbf{3}$	$\mathbf{1}$	$+2/3$	$+1/3$
d_R^i	$\mathbf{3}$	$\mathbf{1}$	$-1/3$	$+1/3$
ℓ_L^i	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	-1
ν_R^i	$\mathbf{1}$	$\mathbf{1}$	0	-1
e_R^i	$\mathbf{1}$	$\mathbf{1}$	-1	-1
H	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	0
Φ	$\mathbf{1}$	$\mathbf{1}$	0	$+2$

Table 1: Particle contents. In addition to the SM particle contents, the right-handed neutrino ν_R^i ($i = 1, 2, 3$ denotes the generation index) and a complex scalar Φ are introduced.

Under the hypothesis of the classical conformal invariance of the model, the classical scalar potential is described as

$$V = \lambda_H (H^\dagger H)^2 + \lambda (\Phi^\dagger \Phi)^2 + \lambda' (\Phi^\dagger \Phi)(H^\dagger H). \tag{10}$$

Note that when λ' is negligibly small, the SM Higgs sector and the Φ sector relevant for the $B-L$ symmetry breaking are approximately decoupled. If this is the case, we can separately analyze these two Higgs sectors. When the Yukawa coupling Y_N is negligible compared to the $U(1)_{B-L}$ gauge coupling, the Φ sector is the same as the original Coleman-Weinberg model [2], so that the radiative $U(1)_{B-L}$ symmetry breaking will be achieved. Once Φ develops its VEV, the tree-level mass term for the Higgs is effectively generated through the third term in Eq. (10). Taking λ' negative, the induced mass squared for the Higgs doublet is negative and as a result, the electroweak symmetry breaking is driven in the same way as in the SM. In this letter, we show that in the $B-L$ extended SM with the classical conformal symmetry, such a simple symmetry breaking is realized in a wide range of the parameter space of the model which is consistent with both the theoretical and experimental requirements.

$B-L$ **symmetry breaking**.— Assuming a negligible λ', let us first investigate the radiative $B-L$ symmetry breaking. We employ the renormalization group (RG) improved effective potential at the one-loop level of the form [16]

$$V(\phi) = \frac{1}{4}\lambda(t) G^4(t) \phi^4, \tag{11}$$

where $\phi/\sqrt{2} = \Re[\Phi]$, $t = \log[\phi/M]$ with the renormalization point M, and

$$G(t) = \exp\left[-\int_0^t dt'\ \gamma(t')\right] \tag{12}$$

with the anomalous dimension (in the Landau gauge) explicitly described as

$$\gamma = \frac{1}{32\pi^2}\left[\sum_i (Y_N^i)^2 - a_2 g_{B-L}^2\right]. \tag{13}$$

Here, g_{B-L} is the $B-L$ gauge coupling, and $a_2 = 24$. Renormalization group equations for coupling parameters involved in our analysis are listed below:

$$\begin{aligned}
2\pi\frac{d\alpha_{B-L}}{dt} &= b\alpha_{B-L}^2, \\
2\pi\frac{d\alpha_\lambda}{dt} &= a_1\alpha_\lambda^2 + 8\pi\alpha_\lambda\gamma + a_3\alpha_{B-L}^2 - \frac{1}{2}\sum_i(\alpha_N^i)^2, \\
\pi\frac{d\alpha_N^i}{dt} &= \alpha_N^i\left(\frac{1}{2}\alpha_N^i + \frac{1}{4}\sum_j \alpha_N^j - 9\alpha_{B-L}\right),
\end{aligned} \tag{14}$$

where $\alpha_{B-L} = g_{B-L}^2/(4\pi)$, $\alpha_\lambda = \lambda/(4\pi)$, $\alpha_N^i = (Y_N^i)^2/(4\pi)$, and the coefficients in the beta functions are explicitly given as $b = 12$, $a_1 = 10$ and $a_3 = 48$.

Setting the renormalization point to be the VEV of ϕ at the potential minimum, $\phi = M$ or equivalently $t = 0$, the stationary condition,

$$\left.\frac{dV}{d\phi}\right|_{\phi=M} = \left.\frac{e^{-t}}{M}\frac{dV}{dt}\right|_{t=0} = 0, \tag{15}$$

leads to one condition among coupling constants at the potential minimum such that

$$\frac{d\alpha_\lambda}{dt} + 4\alpha_\lambda(1-\gamma) = \frac{1}{2\pi}\left(10\alpha_\lambda^2 + 48\alpha_{B-L}^2 - \frac{1}{2}\sum_i(\alpha_N^i)^2\right) + 4\alpha_\lambda = 0 \tag{16}$$

at $t = 0$. For coupling values well within the perturbative regime, $\alpha_\lambda \sim \alpha_{B-L}^2 \sim (\alpha_N^i)^2 \ll 1$, we find a solution

$$\alpha_\lambda(0) \simeq -\frac{6}{\pi}\left(\alpha_{B-L}(0)^2 - \frac{1}{96}\sum_i(\alpha_N^i(0))^2\right). \tag{17}$$

In this approximation, it is straightforward to obtain the SM singlet Higgs boson mass of the form,

$$m_\phi^2 = \left.\frac{d^2V}{d\phi^2}\right|_{\phi=M} = \left.\left(\frac{e^{-t}}{M}\frac{d}{dt}\right)^2 V\right|_{t=0} \simeq -16\pi\alpha_\lambda(0)M^2 \tag{18}$$

under the conditions of Eq. (17) and $\alpha_\lambda \sim \alpha_{B-L}^2 \sim (\alpha_N^i)^2 \ll 1$. Therefore, when we choose the coupling constant to be $\alpha_\lambda(0) < 0$, the effective potential has a minimum at $\phi = M$ and

the $B-L$ symmetry is radiatively broken[1]. Note that in the limit $Y_N^i \to 0$, the system is the same as the one originally investigated by Coleman-Weinberg [2], where the U(1) gauge interaction plays the crucial role to achieve the radiative symmetry breaking keeping the validity of perturbation. In this sense, gauging the $\mathrm{U}(1)_{B-L}$ is necessary although it is not required for the purpose to implement the seesaw mechanism.

When $\alpha_N^i \ll \alpha_{B-L}$ and α_N^i is neglected, the analytic solutions of the RGEs in Eq. (14) are known [2]:

$$\begin{aligned} \alpha_{B-L}(t) &= \frac{\alpha_{B-L}(0)}{1-\frac{b}{2\pi}\alpha_{B-L}(0)t}, \\ \alpha_\lambda(t) &= \frac{a_2+b}{2a_1}\alpha_{B-L}(t) + \frac{A}{a_1}\alpha_{B-L}(t)\tan\left[\frac{A}{b}\ln\left(\frac{\alpha_{B-L}(t)}{\pi}\right)+C\right], \end{aligned} \tag{19}$$

where $A = \sqrt{a_1 a_3 - (a_2+b)^2/4}$, and C is the integration constant fixed to satisfy Eq. (16). Using these solutions, the RG improved effective potential is given by

$$V = \pi \frac{\alpha_\lambda(t) M^4}{\left(1-\frac{b}{2\pi}\alpha_{B-L}(0)t\right)^{a_2/b}} e^{4t}. \tag{20}$$

We show the effective potential in Fig. 1 for $\alpha_{B-L} = 0.01$ as an example. Here $\alpha_\lambda(0) = -1.91 \times 10^{-4}$ is fixed to satisfy Eq. (16), and the effective potential has its minimum at $\phi/M = 1$.

The exact solution of $\alpha_\lambda(t)$ in Eq. (19) becomes singular at some infrared region and thus the instability of the effective potential occurs [17]. For $\alpha_{B-L}(0) = 0.01$, for example, we find that this instability occurs at a very infrared point, $t \simeq -41$, and it is hard to recognize this point in Fig. 1. In a phenomenological point of view, this infrared point is far below the dynamical scale of the QCD and we cannot trust the effective potential from perturbative calculations. In this letter, we do not discuss this instability issue, but it is interesting to note a connection of this instability with the negativeness of $\alpha_\lambda(0)$. The latter implies that the running coupling constant crosses zero at some scale and the classical potential vanishes if we choose the renormalization scale there. This is the condition for the perturbative validity of the Coleman-Weinberg scenario [18, 16].

An extra gauge boson associated with the $\mathrm{U}(1)_{B-L}$ gauge symmetry, the so-called Z' boson, acquires its mass through the $B-L$ symmetry breaking, $m_{Z'} = 2g_{B-L}M$. As is well-known, there is a relationship between the Z' boson mass and the SM singlet Higgs boson mass in the

[1]It may seem strange that the coupling constant $\alpha_\lambda(0)$ must be taken negative for the extremum to be locally stable with a positive mass squared. However the true effective coupling constant at the minimum should be defined as $\lambda_{eff} = V^{(4)}|_{t=0}/6$, which is related to $\lambda(0)$ as $\lambda(0) = -3\lambda_{eff}/22$ and hence positive. Then the ϕ boson mass is given by the same formula as in (4).

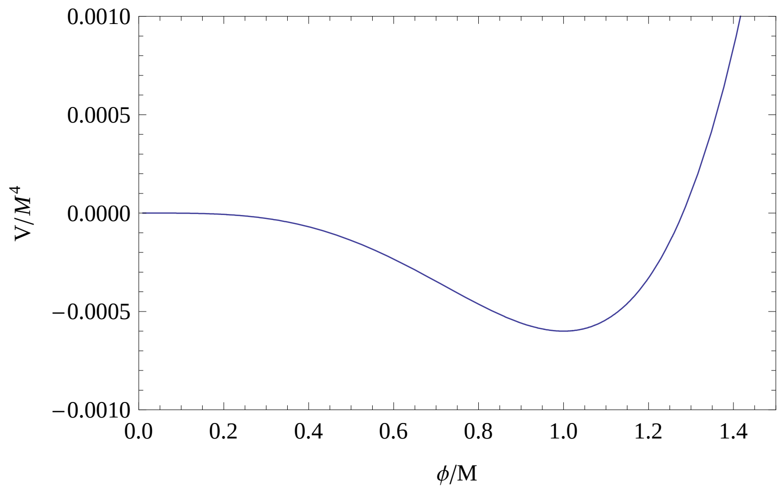

Figure 1: The RG improved effective potential. Here, we have taken $\alpha_{B-L}(0) = 0.01$, accordingly $\alpha_\lambda(0) = -1.91 \times 10^{-4}$ and $C = 1.27$ are fixed so that the effective potential has its minimum at $\phi/M = 1$.

Coleman-Weinberg model [2]. Neglecting α_N^i in Eqs. (17) and (18), we find the mass relation

$$\left(\frac{m_\phi}{m_{Z'}}\right)^2 \simeq \frac{6}{\pi}\alpha_{B-L}. \tag{21}$$

The hierarchy between the two masses is a general consequence of the Coleman-Weinberg model where the symmetry breaking occurs under the balance between the tree-level quartic coupling and the terms generated by quantum corrections. The scalar boson ϕ can be much lighter than the Z' gauge boson and possibly comparable with the SM Higgs boson. Then, as we discuss later, the two scalars mix each other.

Eqs. (17) and (18) suggest that as the Yukawa coupling α_N becomes larger, the SM singlet Higgs boson mass squared is reducing and eventually changes its sign. Therefore, there is an upper limit on the Yukawa coupling in order for the effective potential to have the minimum at $\phi = M$. This is in fact the same reason as why the Coleman-Weinberg mechanism in the SM Higgs sector fails to break the electroweak symmetry when the top Yukawa coupling is large as observed. Analyzing the RG improved effective potential with only one Yukawa coupling α_N, the SM singlet Higgs boson mass as a function of the Yukawa coupling is depicted in Fig. 2. The minimum at M in the effective potential changes into the maximum for $\alpha_N(0) > 9.8\alpha_{B-L}(0)$.

Electroweak symmetry breaking.— Now let us consider the SM Higgs sector. In our model, the electroweak symmetry breaking is achieved in a very simple way. Once the $B-L$ symmetry is broken, the SM Higgs doublet mass is generated through the mixing term between

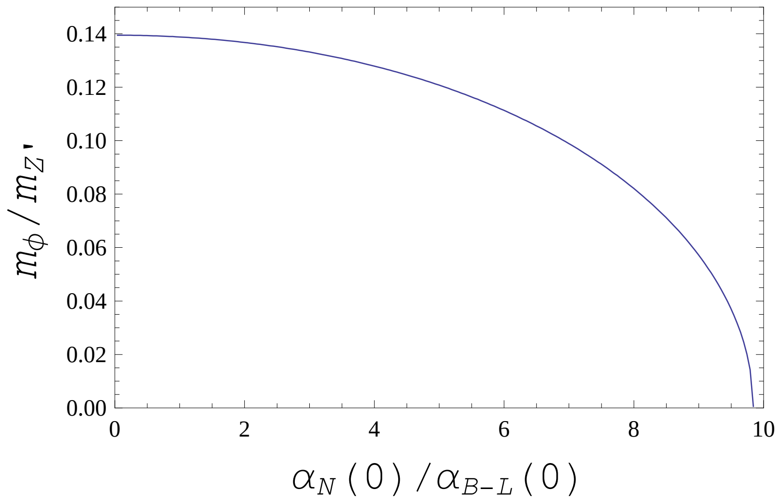

Figure 2: The SM singlet Higgs boson mass as a function of the Yukawa coupling. Here we have taken $\alpha_{B-L}(0) = 0.01$ and accordingly, fixed $\alpha_\lambda(0)$ to satisfy the stationary condition in Eq. (16). For $\alpha_N(0) \simeq 9.8\alpha_{B-L}(0)$, the potential minimum at $\phi = M$ changes into the maximum.

H and Φ in the scalar potential (see Eqs. (10) and (5)),

$$\mu^2 = \frac{\lambda'}{2} M^2. \tag{22}$$

Choosing $\lambda' < 0$, the electroweak symmetry is broken in the same way as in the SM. However, the crucial difference from the SM is that in our model, the electroweak symmetry breaking originates form the radiative breaking of the $\mathrm{U}(1)_{B-L}$ gauge symmetry. At the tree level the Higgs boson mass is given by $m_h^2 = 2|\mu^2| = |\lambda'|M^2 = 2\lambda_H v^2$ where $\langle h \rangle = v = 246$ GeV. Note that the mass is independent of λ' when it is written in terms of the VEV of h. If λ' is sufficiently small, the mass formula for the Higgs boson is given by Eq.(6). Then, by imposing the triviality (up to the Planck scale) and the vacuum stability bounds, the Higgs boson mass is given in a range 130 GeV$\lesssim m_h \lesssim$ 170 GeV as in the SM [13]. When we include the effect of the mixing with the $B-L$ sector, the Higgs boson mass has a correction (see Eq.(8))

$$\Delta m_h^2 = -\frac{64}{3} B_\phi v^2, \tag{23}$$

where B_ϕ is the 1-loop coefficient of the ϕ field contribution to the Higgs potential;

$$B_\phi = \frac{\lambda'^2}{128\pi^2}. \tag{24}$$

The correction is negligible if we take $|\lambda'|$ as a phenomenologically favorable value (discussed later), e.g. $\lambda' \sim 0.002$ for $M \sim 3$ TeV. Since $|\lambda'| = 2\lambda_H (v/M)^2$, it becomes smaller for larger

M. The effect of the right-handed neutrinos to the coefficient B is also roughly given by $Y_D^4/(16\pi^2)$, which can be sizable when the right-handed neutrino mass scale is higher than 10^{13} GeV [19, 20]. In our case, the effect is negligible since the right-handed neutrino mass scale should be small $\ll 10^{13}$ GeV as will be discussed later. Another effect of the right-handed neutrinos to the coefficient λ' is also discussed later.

Validity of the approximation.— We have discussed the $B-L$ symmetry breaking sector and the SM Higgs sector separately and in this case, both the symmetry breakings are easily realized. This treatment should be justified only in a limited parameter space of the model, but we will see that the validity of this approximation can be held easily once we impose a phenomenologically viable condition for the $B-L$ symmetry breaking scale $M >$ a few TeV.

First, we determine the upper bound of the $B-L$ gauge coupling. We require all the coupling constants in our model are in the perturbative regime below the Planck scale. Because of its large beta function coefficient, the $B-L$ gauge coupling is severely constrained by this condition. We can find that $\alpha_{B-L}(0) \lesssim 0.015$ is required if we take its renormalization scale at $M \gtrsim 3$ TeV. (The experimental lower bound for the Z' gauge boson requires M to be larger than this value as discussed later.)

Now we will investigate the upper bound for the coupling constant λ' in order to satisfy the separability condition for the $B-L$ and the electroweak sectors. Using ϕ as the background field for H, the negative Higgs doublet mass squared is induced and as a result, the electroweak symmetry is broken. Once the Higgs doublet develops a VEV, ϕ is mixed with the Higgs field h through the coupling λ'. Around the minimum ($h = v, \phi = M$), the condition that the mixing term does not drastically deform the potential of ϕ with a curvature m_ϕ^2 is given by

$$|\lambda'|Mv \ll m_\phi^2. \tag{25}$$

Neglecting Y_N^i, again, in Eq. (18), for simplicity, we find

$$|\lambda'| \ll 96\alpha_{B-L}^2 \left(\frac{M}{v}\right). \tag{26}$$

For example, if we take $\alpha_{B-L} = 0.01$ and $M = 3$ TeV, the condition becomes $|\lambda'| \ll 0.12$. As M becomes larger, the constraint on λ' becomes looser.

Another condition we need to check is the smallness of the λ' contribution to the beta function of α_λ, which we have neglected. Non-zero λ' contributes to the beta function of λ and adds a term $\alpha_{\lambda'}^2$ in the right hand side of Eq. (14), where $\alpha_{\lambda'} = \lambda'/(4\pi)$. In order not to change our analysis of the scalar potential of ϕ, let us require

$$\alpha_{\lambda'}^2 \ll 48\alpha_{B-L}^2. \tag{27}$$

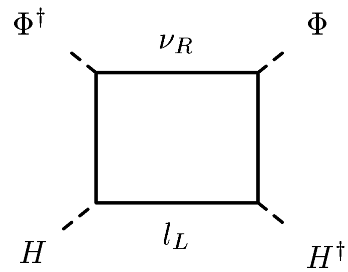

Figure 3: One-loop diagram inducing the mixing term $(\Phi^\dagger\Phi)(H^\dagger H)$ through the right-handed neutrinos.

We find $|\lambda'| \ll 0.87$ for $\alpha_{B-L} = 0.01$. It is looser than the first condition.

The final, but the most stringent condition for λ' comes from the Higgs boson mass bounds. The SM Higgs potential with the induced Higgs doublet mass term in Eq. (22) leads to the relation

$$|\lambda'| = \left(\frac{m_h}{M}\right)^2, \tag{28}$$

at the electroweak symmetry breaking vacuum, where m_h is the physical Higgs boson mass. Since the electroweak symmetry breaking is triggered by the negative mass term, the dynamics is controlled by the classical action in Eq. (5). Therefore, the triviality and the vacuum stability bounds in the SM [13] require the Higgs boson mass in the range 130 GeV$\lesssim m_h \lesssim$ 170 GeV. Since for the phenomenologically favorable value of $M \gtrsim 3$ TeV, the contribution of λ' to the RGE evolution of the Higgs quartic coupling λ_H is negligible, we can apply this Higgs boson mass bound in the SM to constrain the range of λ' such as $(130\mathrm{GeV}/M)^2 \lesssim |\lambda'| \lesssim (170\mathrm{GeV}/M)^2$. This bound on λ' for $M \gtrsim 3$ TeV becomes $0.0019 \lesssim |\lambda'| \lesssim 0.0032$ and is consistent with those evaluated above for $\alpha_{B-L} = 0.01$. The larger value of M reduces λ' and the two sectors can be further treated separately.

Scale of the $B - L$ breaking.— The energy scale of the $B - L$ symmetry breaking is constrained from both of the above and the below by the stability of the electroweak scale and the experimental bound for the Z' gauge boson mass.

We have imposed the classical conformal invariance and the absence of the quadratic divergences to solve the gauge hierarchy problem, but if the $B-L$ symmetry breaking scale is large, we may need a fine tuning of the electroweak scale again to cancel the radiative corrections by some heavy states associated with the $B - L$ symmetry breaking[2]. At the one-loop level (see Fig. 3), the mixing term $(\Phi^\dagger\Phi)(H^\dagger H)$ can be also induced through the right-handed neutrinos with the Yukawa interactions in Eq. (9). Substituting $\phi = M$, we obtain the correction to the

[2] A similar discussion is given in [21].

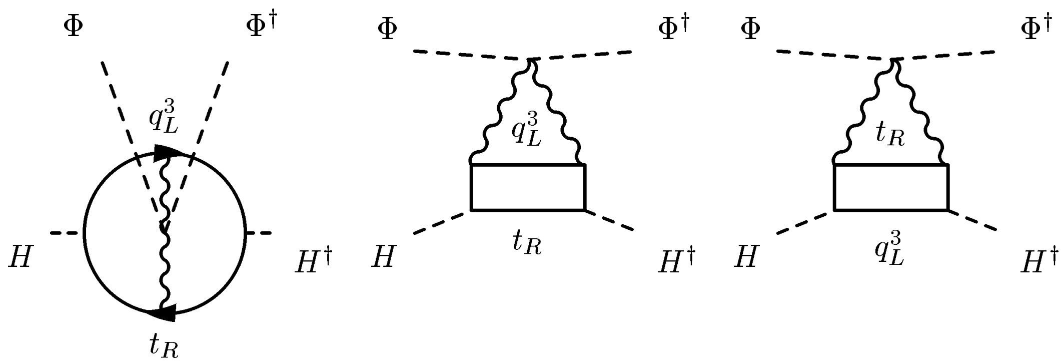

Figure 4: Two-loop diagrams inducing the mixing term $(\Phi^\dagger\Phi)(H^\dagger H)$ through the top-quarks and the $B-L$ gauge bosons. The wavy lines represent the propagators of the $B-L$ gauge bosons.

effective Higgs boson mass squared such as

$$\Delta m_h^2 \sim \frac{Y_D^2 Y_N^2}{16\pi^2} M^2 \sim \frac{m_\nu M_N^3}{16\pi^2 v^2}, \tag{29}$$

where we have used the seesaw formula, $m_\nu \sim Y_D^2 v^2/M_N$ with $M_N = Y_N M$. For the stability of the electroweak vacuum, Δm_h^2 should be smaller than the electroweak scale[3]. Thus, we can obtain the upper bound of M_N once m_ν is fixed. For example, when the neutrino mass is around $m_\nu \sim 0.1$ eV, there is an upper bound for the Majorana mass $M_N \lesssim 10^7$ GeV and hence $M \lesssim 10^7/Y_N$ GeV. As we discussed before, the Yukawa coupling Y_N cannot be larger than the $B-L$ gauge coupling g_{B-L} in order to realize the radiative breaking in the $B-L$ sector.

We also consider higher order corrections to the mixing term, which are dominated by two-loop diagrams involving top-quarks and the $B-L$ gauge boson (see Fig. 4). Again, substituting $\phi = M$, we obtain the correction such as

$$\Delta m_h^2 \sim Y_t^2 \left(\frac{\alpha_{B-L}}{4\pi}\right)^2 M^2, \tag{30}$$

where $Y_t \sim 1$ is the top quark Yukawa coupling. This correction leads to the upper bound $M \lesssim 10^5$ GeV $\times$ $(0.01/\alpha_{B-L})$, which is severer than the correction given by the right-handed neutrino. If there exist other mass scales, additional radiative contributions to the Higgs boson mass can be generated. Thus, in order to ensure the stability of the electroweak scale, we implicitly assume that there exists no other new physics between the $B-L$ breaking scale and the Planck scale.

[3] Note that the condition is equivalent to an issue of the *naturalness* of the λ' coupling. The dynamics of the SM Higgs sector is still governed by the classical potential with a negative mass squared, and hence the Higgs boson mass bounds are not modified by this type of the corrections.

On the other hand, there is an experimental lower bound for the scale M. The Z' boson with mass $m_{Z'} = 2g_{B-L}M$ couples to all the SM fermions as well as the right-handed neutrinos through the $B-L$ charges. The experimental search for the Z' boson at LEP II gives the limit on the $B-L$ symmetry breaking scale $M \gtrsim 3$ TeV [22]. This bound is consistent with the bound from the direct search of the Z' boson at Tevatron [23].

Conclusions and discussions.— The hypothesis of the classical conformal invariance may solve the gauge hierarchy problem in the Standard Model [1]. Once this hypothesis is imposed on gauge theories, the gauge symmetry can be radiatively broken in the Coleman-Weinberg potential. Unfortunately, this scenario cannot work in the Standard Model since the large top Yukawa coupling destabilizes the effective Higgs potential and hence, some extensions of the model are necessary. Motivated by the recent observations of neutrino masses and flavor mixings, we consider the minimal $B-L$ extension of the Standard Model, in which the right-handed neutrinos of each generation are necessarily introduced to cancel the gauge anomalies and tiny neutrino masses can be naturally explained by the seesaw mechanism. Under the hypothesis of the classical conformal invariance, we have investigated the radiative gauge symmetry breaking in this model. Taking the mixing parameter between the SM Higgs and the SM singlet scalars to be small, we have analyzed the effective potential of the Higgs and the singlet sectors separately and proposed a very simple realization of both the $B-L$ and the Standard Model gauge symmetry breakings. We have found that this scenario can work for a wide range of parameter space keeping the theoretical and phenomenological requirements.

Finally, we comment on phenomenological aspects of our model. If the $B-L$ symmetry breaking scale is around its experimental lower bound $M \simeq 3$ TeV, all new particles in the $B-L$ extended SM, the Z' boson, the right-handed neutrinos and the SM singlet Higgs boson, can be as light as a few TeV. There has been a number of works of Large Hadron Collider (LHC) phenomenologies on the production of such new particles [24]. In particular, the Z' boson can be produced through Drell-Yang processes and once produced, its decay into di-leptons with a large branching ratio would provide us clean signatures.

After the electroweak symmetry breaking, the SM singlet Higgs boson mixes with the SM Higgs boson. In our model the SM singlet Higgs boson is relatively light to the Z' boson (see Eq. (21)) and its mass can be as low as a few hundred GeV. When the singlet Higgs boson is light, the mixing between two Higgs bosons can be sizable and affect the SM Higgs boson phenomenology.

The leptogenesis [25] through the lepton number and CP violating decays of the right-handed Majorana neutrino is a very simple mechanism for baryogenesis. In normal thermal leptogenesis scenario, there is a lower mass bound on the lightest right-handed neutrino, $M_N \gtrsim 10^9$ GeV

[26], in order to achieve the realistic baryon asymmetry of the present universe. In our model, as discussed above, the theoretical requirement constrains the right-handed neutrino mass to be smaller than this bound, for example, $M_N \lesssim Y_N \times 10^5$ GeV for $\alpha_{B-L} \sim 0.01$. However, the leptogenesis is still possible through the resonant leptogenesis [27] when right-handed neutrino masses are well-degenerated. One interesting feature in our model is that at high temperature, the right-handed neutrinos can be in thermal equilibrium with the SM particles through the $B-L$ gauge interaction, so that a large efficiency factor can be easily obtained.

The existence of (cold) dark matter is strongly supported by various observations of the present universe and suggests the need of extending the SM since the SM includes no suitable candidate for dark matter. A simply extended model has been proposed [28, 29], where only a SM singlet scalar with an odd parity is introduced. This scalar can be a suitable candidate for dark matter through the coupling with the SM Higgs boson. It is easy to extend our model in the same way, i.e., we introduce a parity odd scalar which is singlet under both the SM and $B-L$ gauge groups. According to our hypothesis of the classical conformal invariance, this singlet scalar has no mass term. There are only two terms for the interaction among the dark matter and the other particles at the renormalizable level,

$$\mathcal{L} \supset -\lambda_1 (H^\dagger H)\chi^2 - \lambda_2 (\Phi^\dagger \Phi)\chi^2, \tag{31}$$

where χ is the dark matter candidate. The mass for the dark matter is generated by the $B-L$ and electroweak symmetry breakings, $m_\chi^2 = \lambda_1 v^2 + \lambda_2 M^2$, together with the interaction terms among the dark matter and the Higgs bosons,

$$\mathcal{L}_{int} \supset -\frac{\lambda_1}{2}(2vh + h^2)\chi^2 - \frac{\lambda_2}{2}(2M\phi + \phi^2)\chi^2, \tag{32}$$

which plays the essential role in the dark matter annihilation processes in the early universe. When we neglect λ_2, this model is essentially the same as the one discussed in [29]. Thanks to the absence of the dark matter mass term, dark matter physics is controlled by only two unknown parameters, m_χ and m_h, which are well constrained by the observed dark matter relic density [29]. The couplings λ_1 and λ_2 are involved in the RGEs of quartic Higgs couplings and potentially affect the SM Higgs boson mass bound from triviality (up to the Planck scale) and vacuum stability arguments. We leave this study for future works.

Acknowledgments

We would like to thank Hajime Aoki, Eung Jin Chun and Kazuo Fujikawa for useful discussions and comments. This work of N.O. is supported in part by the Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture of Japan, No. 18740170.

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